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Endogenous Fluctuations in the Demand for Education

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Abstract

Enrollment rates to higher education reveal quite large variation over time that probably cannot be explained by productivity shocks alone. We develop a human capital model with overlapping generations that creates endogenous fluctuations in the demand for education. Agents are heterogenous in their beliefs about future wage differentials. Costly access to information on the returns to education induces agents to use potentially destabilizing backward looking prediction rules. Only if previous generations experience losses from human capital investment, agents will choose a more sophisticated prediction rule that dampens the cycle. Access to information becomes key for stable flows to higher education.

Keywords: expectations, human capital investment, endogenous fluctuations, inter-generational spill-overs

1 Introduction

Enrollment rates to higher education vary over time. This holds true for a many countries and educational fields. In the U.S. college enrollment rates fluctuated between 37% and 47% in the years from 1968 to

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1988. Controlling for individual and regional effects does not cancel out the dynamics (Dellas and Sakellaris 1995). Time series for Sweden indicate a variation of college enrollment rates of 20 to 24 year old men between 34% and 46% in the years from 1963 to 1991 (Topel 1997). For engineering degrees at universities, Germany was confronted with enrollment rates between 8% to 12% of all graduates from upper secondary schooling from 1975 to 1998. For those students enrolling to non-university type of higher education rates changed between 32% and 52% of all graduates with a specialized upper secondary degree (Neugart 2001). Freeman (1976b) shows for various fields of study the dynamics of enrollments to U.S. colleges.[1]

There is a large literature aiming to explain changes in the demand for higher education with variations in expected relative wages. Approaches taken so far may be divided according to the way how agents' expectations are modelled. Backward looking expectations postulate that students entering higher education programs form expectations on future relative wages using past experiences. What counts in the 'cost-benefit' analysis that underlies the human capital investment decision is actual wages or most recently paid wages relative to what could be earned without investment into schooling. There is time series evidence, usually on the basis of dynamic regression models, that finds enrollment rates driven by backward looking expectations (Freeman 1975a and 1975b, Freeman 1976a, Borghans et al. 1996, Duchesne and Nonneman 1998, Quinn and Price 1998, Card and Lemieux 2000). On the other hand, one may employ rational expectations. This assumes that agents can make correct forecasts on future relative wages. Zarkin (1983, 1985) and Siow (1984) have estimated rational expectations models for enrollments of teachers and lawyers, respectively. Both find support for the hypothesis of rationally forecasting agents.

Reviewing the literature on the demand for education Freeman (1986) concludes, that, when assuming backward looking expectations, the internal market structure can generate cobweb type of 'ups' and 'downs' in enrollment rates. However, the estimated models would imply damped oscillations. To arrive at continued oscillations one would have to recur to large shocks. Such shocks are also needed if one wants to explain cyclical behavior in enrollment rates under the

[1] A collection of data on enrollments for a large range of countries can be found on the homepage of the UNESCO *http://unescostat.unesco.org/en/stats/stats0.htm*. The compendium by Titze et al. (1986, 1993) contains time series on enrollments for all major subjects for Germany going back to 1820. The fluctuations shown there are quite striking.

assumption of rational expectations. Clearly, both approaches shift the attention to factors that lie outside the labor market.

The aim of our paper is to offer an endogenous explanation of enrollment dynamics. Even though we focus on internal forces, we do not think that exogenous shocks shifting the demand side of the labor market should be ruled out as a potential cause. However, the role of exogenous shocks may be less important than needed to explain continuing large fluctuations in enrollments.

We develop a human capital investment model with overlapping generations. Agents, deciding on whether to invest into schooling use either a costless backward looking predictor or have costs for making a more sophisticated prediction on future relative wages. Here, we borrow from the finance literature (see for example Brock and Hommes 1998). The predicted wage rate then determines the schooling decision. The other choice, the one on the predictor, is driven by the performance of backward looking agents of the previous generation. Inter-generational spill-overs guide the new born generation in their choice of the predictor rule. They are more likely to predict with a sophisticated mechanism if the previous generation did poorly with the backward looking predictor. The dynamics of the model are such that backward looking expectations may destabilize enrollments, very much in the the spirit of an unstable cobweb. However, oscillations will be bounded as agents switch to a forward looking predictor if the previous, mostly naive population, regrets its schooling decision.

The notion of not so rational agents in our model relates to survey evidence for European and U.S. college students on studying motives (Brunello, Lucifora and Winter-Ebmer 2001, and Betts 1996, respectively). The former find, based on survey data of more than 6000 college students, that the average expected college wage gains are larger than estimated actual wage gains. In their study overoptimistic students expect to earn far more than 10% than they actually will. The latter study with U.S. data also finds overestimation of wage gains from college attendance of approximately 10%. Both results give hints that the enrollment decision is not based on a rational forecast. However, as it is evidence for a specific year in each data set only, it does not yield insight into whether beliefs change over time. That gap may be filled with simulation studies as they can be found in (Borghans et al. 1996 and Neugart 2001). Both authors estimate a model for enrollments to higher education based on backward looking expectations. They use the model to generate a time series on hypothetical enrollments. That is enrollments, had students known the actual wage for the time of graduation. Comparing hypothetical and actual enrollments gives information on how many students would have or would have not chosen schooling had they known the future labor market status. All we want to state here with respect to these findings is that the number of students making the 'wrong' decisions varies over time quite tremendously. Certainly, not having the right forecast may be due to frequent shocks to the market. However, those would have to be rather marked.

The inter-generational spill-over effect in the belief formation of our agents can be put into the context of other models where the schooling decision is also a function of the social environment. Externalities so far investigated were either intra- or inter-generational. Bala and Sorger (1998, 2001) study the former in a spatial temporal set-up. There, returns on human capital investments do not only depend on the agent's effort but also on the human capital endowment of so called peer groups. Good neighborhoods, or skilled socially relevant fellow agents may have a positive impact on human capital accumulation and vice versa. One important policy implication from such models is that with intra-generational spill-overs one may observe an endogenous formation of skilled and un-skilled agents over space. Removing credit constraints may not shut off stratification. Inter-generational spill-overs play central roles in Orazem and Tesfatsion (1997) and de la Croix (2001). The over-lapping generations model of Orazem and Tesfatsion consists of multiple dynasties. In lack of an adequate predictor on future wages, students learn from their parents whether it pays to invest into schooling. Higher observed returns on education of the parents induces their children to increase effort. Orazem and Tesfatsion study the impact of taxing adults and its impact on the schooling decision of their children. In de la Croix a new born generation inherits aspirations and human capital of the previous generation. While a high human capital level serves as a positive incentive to invest in ones own education, high aspirations carry the opposite sign. The composite effect on the current generations' consumption behavior may be such that consumption is too high given current productivity growth. The new generation may not sufficiently invest into human capital to keep the economy growing. Depending on the relative strength of the two externalities the economy may run into a poverty trap or cycle along a growth path.

Studying the impact of heterogenous beliefs on the schooling decision contributes to the interpretation of estimates on schooling behavior. Manski (1993) already emphasized the lack of evidence that prevailing assumptions on expectations are correct. Moreover he indicated that wrong assumptions on expectations may lead to biases in empirical estimates of schooling choices. The reason is that it is impossible to tell how students make their schooling decisions from observing schooling choices as long as one does not know how students perceive the returns to schooling. Assume, that the variable l describes the schooling choice, w the expected returns and the function f maps l on w, $l = f(w)$. Observing the choices and the returns only allows to determine f, if one assumes a specific expectation rule. If the assumption on expectations is false, however, so will be the estimates of the schooling choice. Clearly, a better understanding of how expectations are formed is required.

There is at least one aspect where our study carries policy relevance. Large variations in enrollments may lead to situations where universities run into capacity constraints which will very likely reduce the quality of education. Holding excess capacity is a costly policy option. Knowing about the underlying forces of enrollment dynamics may guide policy makers to more efficient solutions.

Our paper is organized in the following way. We first sketch a general model with respect to consumers, firms and belief formation when agents live for two periods and generations overlap. The next section develops a more specific model. It is assumed that agents choose between a naive and a rational predictor. A numerical example is introduced for which the dynamic properties are studied analytically and with computational methods. Section 4 presents a variation on the theme. Here, we substitute rationally predicting agents with steady state forecasters – agents that know the long run wage differential between high-skill and low-skill work – and redo our analysis. Section 5 concludes.

2 The Model

2.1 The firms

There are two production sectors in this economy, sector H and sector L, producing the same commodity. Sector H employs high-skilled labor, sector L employs low-skilled labor. We assume that both sectors use labor as the only factor of production. Firms in sector L

produce according to a constant returns to scale production technology, implying a constant real wage rate for low-skilled labor which we normalize to unity. Demand for low-skilled labor is therefore perfectly elastic at this real wage of 1. Firms in sector H produce according to a concave production technology $f(l)$. Profit maximization yields a decreasing demand function for high-skilled labor $l^d(w_t)$ as the solution to $f'(l) = w_t$, where w_t denotes the real wage rate for high-skilled labor in period t.

2.2 The consumers

We assume an overlapping generations structure, where in each period t a continuum of agents of mass one is born that lives for two periods. Agent i has private costs of effort e_i for investing into education. These effort costs are distributed according to some distribution function F with support $[0, 1]$. Agents have one time unit in each period of their life. They can use part of their time endowment in the first period of their life for investing into education. If they invest into education they acquire certain skills which allow them to work in sector H in the second period of their life. If they do not invest in education they will have to work in sector L all of their life. The choice whether or not to invest into education is made in the beginning of the first period of their life, right after they observe the market clearing wage rates in sectors H and L.

An agent born in period t has a lifetime utility function

$$U\left(c_t, c_{t+1}\right),$$

where c_t and c_{t+1} denote consumption in period t and $t+1$, respectively. We will assume that this utility function is monotonic, strictly concave and twice differentiable. Furthermore, we assume that marginal utility of consumption in the first period approaches infinity as consumption in the first period approaches zero, $\lim_{c_t \to 0} \frac{\partial U}{\partial c_t} = \infty$.

If agent i decides to invest in education his consumption levels are $c_t = 1 - e_i$ and $c_{t+1} = w_{t+1}$. Effort costs can therefore be interpreted as the fraction of time that a young agent has to spend on education to acquire the skills for working in the high-skill sector in the second period of his life. If an agent does not invest in education and decides to work in sector L all of his life, he incurs no effort costs. His consumption will then be $c_t = c_{t+1} = 1$. Notice that we assume that agents cannot transfer income from one period to the next.

Agent i will therefore invest in education if he expects lifetime utility to be at least as large as when he would not be investing into education. Hence, investment into human capital will occur if and only if

$$U\left(1-e_i, w_{t+1}^e\right) \geq U(1,1),$$

where w_{t+1}^e refers to expectations, formed in period t, on real wages to be paid in sector H in $t+1$. By equating these utility levels we find the *marginal effort level*, that is the effort level for which an agent is indifferent between investing and not investing into human capital. Denote this marginal effort level by $e_t^* = e\left(w_{t+1}^e\right)$. Note that, by our assumption on marginal utility, this marginal effort level always exists for $w_{t+1}^e > 1$. Since utility is decreasing in the effort level all agents with $e_i < e\left(w_{t+1}^e\right)$ will invest in schooling. As the effort level is distributed on $[0,1]$, the fraction of the generation born at time t, with wage expectation w_{t+1}^e, that invests in education will be equal to $F\left(e\left(w_{t+1}^e\right)\right)$. Notice that $e\left(w_{t+1}^e\right)$ as well as $F\left(e\left(w_{t+1}^e\right)\right)$ are upward sloping in w_{t+1}^e as $U(1-e, w)$ is downward sloping in e and upward sloping in w.

Clearly, the decision whether to invest in schooling depends upon the expected wage. In this paper, we assume that there are several ways in which agents may predict the wage rate. Let there be K different of these predictors $w_{k,t+1}$, $k = 1, 2, \ldots, K$. Since the information requirements of the different predictors will differ, it seems reasonable to assume that different amounts of information costs have to be paid for these predictors. In particular, the more sophisticated predictors will require more time and effort to implement than very simple forecasts. For example, because job counsellors have to be consulted, or information on labor market forecasts has to be collected and processed. Each member of the population uses one of the predictors. The size of the fraction of the population of consumers using a predictor depends upon the past success of this predictor. Hence, *inter-generational spill-overs* drive the belief formation. In fact, the predictor choice of the new generation will depend on the level of (dis)satisfaction with the forecast methods of agents of the previous generation. Let n_{kt} denote the fraction of the population using predictor k in period t. Clearly, we must have $\sum_{k=1}^{K} n_{kt} = 1$. Assuming that beliefs are independently distributed across the population of consumers, total enrollment in schooling in period t will be given by

$$E_t = E(w_{1,t+1}, w_{2,t+1}, \ldots, w_{K,t+1}) = \sum_{k=1}^{K} n_{kt} F\left(e(w_{k,t+1})\right).$$

We will abstract from dropouts and will assume that everybody that enrolls in schooling in period t, will indeed acquire the necessary skills and supply his labor to the market for high-skilled labor in period $t+1$. Total supply of high-skilled labor in period $t+1$ will therefore be

$$l^s\left(w_{1,t+1}, \ldots, w_{K,t+1}\right) = \sum_{k=1}^{K} n_{kt} F\left(e(w_{k,t+1})\right).$$

2.3 Market equilibrium

The next step is to investigate equilibrium on the market for high-skilled labor. The market clearing condition for period $t+1$ becomes

$$l^d\left(w_{t+1}\right) = \sum_{k=1}^{K} n_{kt} F\left(e\left(w_{k,t+1}\right)\right). \tag{1}$$

Denote by w^* the unique solution to $l^d(w) = F(e(w))$. Furthermore, let $l_w^d \equiv \frac{\partial l^d}{\partial w}(w^*) < 0$ and $l_w^s \equiv F'(e)\, e'(w^*) > 0$ correspond to the slopes of the labor demand curve and the labor supply curve, respectively, evaluated at the steady state. The relative sizes of these derivatives play an important role in the dynamics of the full model, as will become clear shortly. Equation (1) implicitly defines the market equilibrium wage in period $t+1$ as a function of the set of predictors $w_{k,t+1}$ and corresponding fractions n_{kt} as $w_{t+1} = G\left(w_{k,t+1}, \ldots, w_{K,t+1}, n_{1t}, \ldots, n_{Kt}\right)$.[2]

Let us now try to determine whether a solution to the market clearing condition always exists. Notice that a wedge will be driven between enrollments in education in period t and supply of high-skilled labor in period $t+1$ if the realized wage in period $t+1$ is not larger than the wage for low-skilled labor, i.e. when $w_{t+1} \leq 1$. If $w_{t+1} < 1$ even the high-skilled workers will supply their labor to sector L (where the demand for labor is perfectly elastic). Supply of high-skilled labor

[2] If one of the predictors corresponds to rational expectations or perfect foresight (see the next section), $w_{t+1}^e = w_{t+1}$, then the realized market equilibrium wage also turns up in the right hand side of the market equilibrium condition.

will then be 0. If $w_{t+1} > 1$ total supply of high-skilled labor will be $E_t \geq 0$ (which is independent of w_{t+1} if none of the predictors corresponds to rational expectations and upward sloping if some of the agents are rational). Finally, if $w_{t+1} = 1$ high-skilled workers are indifferent between supplying their labor to sector H or sector L. It follows that, given that $l^d(w_{t+1})$ is downward sloping, sufficient conditions for a market equilibrium to always exist are, $f'(0) > 1$, and $\lim_{l \to \infty} f'(l) \leq 1$.

2.4 Information costs and predictor choice

A newborn generation faces the problem of choosing between different predictors. We now consider how agents decide which forecasting rule they should use. Agents from the previous period who have not predicted the wage rate correctly might have made the wrong schooling decision. Therefore they might *regret* their decision. We let this regret correspond to the costs of the predictor. Notice that the probability of making the wrong decision will be lower for more accurate predictors. We assume that agents utilize the experience of the previous generation for the predictor choice. This means that the decision which predictor to choose is always naive. The information flow from the old generation to the newborn agents can be interpreted as learning or, as we are going to call it, *inter-generational spill-overs*. Information relevant for the predictor choice is carried over from generation to generation. One may think of the media spreading this information. Members of the new generation compare costs of using different predictors. On the one hand, these are the costs that accrue from collecting information about future wage rates. Predicting in a more accurate way than just taking the current situation on the labor market as a cheap predictor, may require consulting employment offices or job counsellors, or finding the relevant forecasts of research institutes that have expertise. On the other hand, agents expect to face costs from using a cheaper predictor that possibly does not give the correct information on returns to education. These expected costs of an incorrect schooling decision are approximated by the regret of the previous generation.

For a formal treatment of the inter-generational spill-overs we have to specify the level of regret for members of the previous generation that have used predictor k. All agents i with an effort level for schooling e_i of $e_i \leq e(w_{k,t+1})$, go into schooling. After the realization of w_{t+1} their ex post preferred decision is to enter schooling when $e_i < e(w_{t+1})$. Therefore, as long as expectations do not equal realized wages, $w_{t+1} \neq w_t$, some agents, using predictor k, would have preferred to make another schooling decision. We can distinguish between two cases: *i)* $w_{t+1} < w_{k,t+1}$ and *ii)* $w_{t+1} > w_{k,t+1}$. Consider the first case. Here, all agents with $e(w_{t+1}) < e_i < e(w_{k,t+1})$ will regret their decision to invest in human capital. Had they known the actual wage, they would not have spend effort on schooling. We can measure the size of their regret or dissatisfaction by comparing their actual utility with the utility they would have gotten, had they made the correct decision. Their actual utility will be

$$U(1 - e_i, w_{t+1}).$$

Given that the market wage is lower than they expected, they would have rather chosen not to invest into their human capital and work in the low-skill sector in the second period of their life. That choice would have given them a utility of

$$U(1, 1),$$

which we will refer to as their *potential* utility. The cost of using predictor k for an agent with effort cost $e_i \in (e(w_{t+1}), e(w_{k,t+1}))$ will therefore be the difference between potential and actual utility. That is

$$R(e_i, w_{t+1}) = U(1,1) - U(1 - e_i, w_{t+1}).$$

It follows from $e(w_{t+1}) < e < e(w_{k,t+1})$ that $R(e_i, w_{t+1}) > 0$. Summing up regret for all agents with predictor k we find aggregate regret $R(w_{k,t+1}, w_{t+1})$, which is

$$R(w_{k,t+1}, w_{t+1}) = \int_{e(w_{t+1})}^{e(w_{k,t+1})} R(e, w_{t+1})\, dF(e). \tag{2}$$

Notice that $R(w_{t+1}, w_{t+1}) = 0$.

Now consider the second case with $w_{t+1} > w_{k,t+1}$. Here, all agents using predictor k with $e(w_{k,t+1}) < e_i < e(w_{t+1})$, who decided to work in the low-skill sector in the second period of their life, will regret their decision not to invest in human capital. They would have invested in human capital had they known the actual wage, which is now higher than they expected. We again consider potential utility, corresponding to investing in schooling, minus actual utility, corresponding to working in sector L, which will be

$$U(1-e_i,w_{t+1})-U\left(1,1\right)=-R\left(e_i,w_{t+1}\right),$$

with $e\left(w_{k,t+1}\right) < e_i < e\left(w_{t+1}\right)$. Summing up regret for all naive agents and swapping the bounds of the integral, we find aggregate regret as

$$R\left(w_{k,t+1},w_{t+1}\right)=\int_{e\left(w_{t+1}\right)}^{e\left(w_{k,t+1}\right)} R\left(e_i,w_{t+1}\right)dF\left(e\right).$$

This expression equals the one of the first case.

Having computed aggregate regret, the final step is to determine the fraction of the newborn population that decides to use the rational predictor. We assume that agents of the newborn generation observe the set of aggregate regrets $R\left(w_{k,t+1},w_{t+1}\right)$ of the old generation. This information may be available from newspapers that report on how satisfied agents of the former generation are with their schooling decision. It may also be transferred to the new generation by socially relevant agents of the older generation. In one way or the other, newborn agents develop a feeling for the extent of regret in the society. They compare aggregate regret, which is their (naive) estimator of their own regret should they choose predictor k, with information costs C_k associated with using predictor k. We suggest the following relationship for the fraction of the population using a certain rule:

$$n_{k,t+1}=H_\beta\left(R\left(w_{1,t+1},w_{t+1}\right)+C_1,\ldots,R\left(w_{K,t+1},w_{t+1}\right)+C_K\right). \quad (3)$$

The parameter β tunes how fast agents respond to differences in costs.

Our full model of human capital investment with heterogeneous beliefs is now given by equations (1) and (3). In the next two sections we will investigate more specific examples of this framework.

3 Rational versus naive predictions

In the literature on demand for education two types of expectation formation processes are typically considered. Usually, human capital models assume *rational expectations* (see for example Becker 1975). Rational expectations imply that agents know how the economy works. Therefore, they can perfectly forecast the wages, i.e. $w_{t+1}^e=w_{t+1}$. On the other hand, many contributions employ *naive expectations* (see

Freeman 1986) where it is assumed that agents predict wages using the most recent wage observation, i.e. $w^e_{t+1} = w_t$. In this section, we try to merge these two approaches, by allowing agents to choose between these two prediction rules. First we will discuss the general setup of this model and then we turn to a numerical example.

3.1 General setup

Let us denote the fraction of rational forecasters in period t as n_t. Hence, $1 - n_t$ corresponds to the fraction of naive forecasters. We furthermore assume that the naive predictor can be obtained for free and that the rational predictor can be obtained at a fixed cost C. The market clearing condition (1) becomes

$$l^d(w_{t+1}) = n_t F(e(w_{t+1})) + (1 - n_t) F(e(w_t)). \tag{4}$$

Let the steady state market equilibrium w^* be defined as the unique solution to $l^d(w) = F(e(w))$. Equation (4) implicitly defines the market clearing wage w_{t+1} as a function of w_t and n_t. Let us denote this by $w_{t+1} = G(w_t, n_t)$. We assume the following specification of the fraction of rational players

$$n_{t+1} = H_\beta(C, R(w_t, w_{t+1})) = H(\beta(R(w_t, w_{t+1}) - C)),$$

where $H'(x) > 0$ and $\lim_{x\to-\infty} H(x) = 0$ and $\lim_{x\to\infty} H(x) = 1$. Here β denotes the intensity of choice which measures how fast agents switch between different predictors. The full model corresponds to the two-dimensional system of first order difference equations:

$$\begin{aligned} w_{t+1} &= G(w_t, n_t), \\ n_{t+1} &= H(\beta(R(w_t, G(w_t, n_t)) - C)). \end{aligned} \tag{5}$$

The following proposition discusses the steady state of this dynamical system and its local stability properties.

Proposition 1 *The steady state of the dynamical system (5) is*

$$(w^*, n^*) = (w^*, H(-\beta C)).$$

This steady state is locally stable if $\left|l^d_w\right| \geq l^s_w$. *If* $\left|l^d_w\right| < l^s_w$, *there exists a critical value* $(\beta C)^*$ *of* βC *such that for* $\beta C < (\beta C)^*$ *the steady* *state is locally stable and for $\beta C > (\beta C)^*$ the steady state is unstable. Furthermore, $(\beta C)^*$ is implicitly given by*

$$H\left(-(\beta C)^*\right) = \frac{l_w^s - \left|l_w^d\right|}{2l_w^s}.$$

Proof. The steady state follows directly from (5) and the observations that, at a steady state, rational and naive agents have correct predictions and regret of naive agents is zero at the steady state. Stability of the steady state is determined by the eigenvalues of the Jacobian matrix evaluated at the steady state. Taking the total differential of (4) and evaluating at the equilibrium (w^*, n^*) gives

$$\left.\frac{\partial G}{\partial w_t}\right|_{(w^*,n^*)} = \frac{(1-n^*)\, l_w^s}{l_w^d - n^* l_w^s} \text{ and } \left.\frac{\partial G}{\partial n_t}\right|_{(w^*,n^*)} = 0.$$

Furthermore, we have

$$\left.\frac{\partial n_{t+1}}{\partial n_t}\right|_{(w^*,n^*)} = \beta H'\left(-\beta C\right) \left.\frac{\partial G}{\partial n_t}\right|_{(w^*,n^*)} = 0.$$

The Jacobian matrix therefore becomes

$$J = \begin{pmatrix} \frac{(1-n^*)l_w^s}{l_w^d - n^* l_w^s} & 0 \\ \left.\frac{\partial n_{t+1}}{\partial w_t}\right|_{(w^*,n^*)} & 0 \end{pmatrix}$$

and has eigenvalues $\lambda_1 = \frac{(1-n^*)l_w^s}{l_w^d - n^* l_w^s}$ and $\lambda_2 = 0$. The steady state is locally stable whenever the eigenvalues lie in the unit circle which, in this case, corresponds to

$$\left(1 - 2n^*\right) l_w^s < \left|l_w^d\right|$$

Clearly, if $\left|l_w^d\right| \geq l_w^s$ this condition is always satisfied. However, if $\left|l_w^d\right| < l_w^s$, then for all $n^* < \frac{l_w^s - \left|l_w^d\right|}{2l_w^s}$, the first eigenvalue is smaller than -1, and the steady state is unstable. ■

This proposition states that if the (absolute value of the) slope of the demand curve is smaller than the slope of the supply curve the steady state might become unstable. Consider what will happen in such a case. First note that in the steady state there is no regret but there are still information costs associated to the rational predictor. Consumers using the naive predictor are therefore better off. Now, if the intensity of choice β is not too large there will always be a sufficient amount of rational agents to stabilize the wage dynamics. However, if β increases, which is accompanied by a rise in the level of competition between different predictors, the equilibrium value of n will go down. This will destabilize the wage dynamics. At the critical value $(\beta C)^*$ of βC, as given in the proposition, a *period-doubling* or *flip bifurcation* occurs and a period two cycle emerges.

For higher values of β even more complicated behavior may occur. Let us briefly discuss the mechanism driving this complicated behavior. Suppose the system starts out close to the unstable steady state. In that case aggregate regret for using the naive predictor will be low and many people from the next generation will use the naive predictor. This destabilizes the wage dynamics and large fluctuations in wages may be observed. Aggregate regret will then go up and subsequently the fraction of rational players in the next generation will increase. This will stabilize the wage dynamics and wages will be driven to their steady state values. Simultaneously, aggregate regret for naive predictors decreases again and we end up close to the initial position. Then the whole story repeats. This mechanism shows that perpetual fluctuations can emerge naturally in a framework with an evolutionary competition between different predictors. In the next subsection we will study a numerical example where this phenomenon is encountered.

3.2 A numerical example

3.2.1 The schooling decision and the labor market

We assume a standard Cobb-Douglas utility function

$$U\left(c_t, c_{t+1}\right) = c_t^{\gamma} c_{t+1}^{1-\gamma}.$$

The marginal effort level e^* is found by equating $U\left(1 - e_t, w_{t+1}^e\right)$ to $U\left(1, 1\right)$. This gives

$$e_t^* = e\left(w_{t+1}^e\right) = 1 - \left(w_{t+1}^e\right)^{-\delta},$$

where $\delta \equiv (1-\gamma)/\gamma$. Furthermore we assume that individual effort costs are uniformly distributed on the unit interval: $F\left(e\right) = e$.

The production technology employed in sector H is given by

$$f_H(l_H) = \frac{\alpha}{\mu} l_H^{\mu},$$

where $\alpha > 0$ is a productivity parameter and $0 < \mu < 1$. Note that for this production technology we indeed have $\lim_{l \to \infty} f'(l) = 0 < 1$ and $\lim_{l \to \infty} f'(l) > 1$ and existence of the market clearing wage is assured. Demand for high-skilled labor in period $t+1$ follows as

$$l^d(w_{t+1}) = (\frac{\alpha}{w_{t+1}})^{\frac{1}{1-\mu}}.$$

Recall that, due to the constant returns to scale technology in sector L, the demand for low skilled labor is perfectly elastic.

Given that the newborn generation can choose between the rational and the naive predictor the equilibrium condition (4) becomes

$$(\frac{\alpha}{w_{t+1}})^{\frac{1}{1-\mu}} = (1 - n_t)(1 - w_t^{-\delta}) + n_t(1 - w_{t+1}^{-\delta}). \tag{6}$$

In order to be able to explicitly solve for w_{t+1} we make the following restriction on the parameters δ and μ: $\delta(1-\mu) = 1$. Note that this requires that $\delta > 1$ (or $\gamma < \frac{1}{2}$, which implies that consumers prefer consumption in the second period of their life over consumption in the first period of their life). If we define $x_t \equiv w_t^{-\delta}$ then (6) is linear in x_{t+1} and can be solved for x_{t+1} as[3]

$$x_{t+1} = G(x_t, n_t) = \frac{1 - (1 - n_t)x_t}{\alpha^{\delta} + n_t}.$$

The steady state equilibrium wage corresponds to the unique solution to $x^* = G(x^*, n_t)$ and is given by $x^* = \left(1 + \alpha^{\delta}\right)^{-1} < 1$, or

[3]Notice that for x_t and n_t satisfying $x_t < \left(1 - \alpha^{\delta} - n_t\right) / (1 - n_t)$ we have $x_{t+1} > 1$, implying $w_{t+1} < 1$. Rational agents will foresee this development and only some of the naive agents will invest in schooling in period t. At a wage rate for high-skilled labor lower than 1, the high-skilled agents prefer to work in sector L. This drives a wedge between enrollment in period t and supply of high-skilled labor in period $t+1$. The supply of high-skilled labor then falls short of demand which will drive up the wage for high-skilled labor. This wage will increase exactly to the point where it equals the wage in the low-skill sector. Therefore, the actual development of x_{t+1} should be written as

$$x_{t+1} = \min\left\{\frac{1 - (1 - n_t)x_t}{\alpha^{\delta} + n_t}, 1\right\}.$$

$w^* = \left(1+\alpha^\delta\right)^{\frac{1}{\delta}} > 1$. Furthermore, $l_w^s = \delta \left(w^*\right)^{-\delta-1}$ and $l_w^d = -\delta\alpha^\delta \left(w^*\right)^{-\delta-1}$, implying that the steady state will be unstable for sufficiently small n^* if $\alpha < 1$.

3.2.2 Information costs and predictor choice

The model is closed by the specification of the predictor choice. For this we need to compute aggregate regret of naive agents (individual and aggregate regret of rational agents will always be zero). Individual regret of naive agents is given by

$$R\left(e_i, w_{t+1}\right) = U\left(1,1\right) - U\left(1-e_i, w_{t+1}\right) = 1 - \left(1-e_i\right)^\gamma w_{t+1}^{1-\gamma}.$$

Aggregate regret becomes

$$\begin{aligned} R\left(w_t, w_{t+1}\right) &= \int_{e(w_{t+1})}^{e(w_t)} R\left(e_i, w_{t+1}\right) dF\left(e\right) \\ &= \int_{1-w_{t+1}^{-\delta}}^{1-w_t^{-\delta}} \left[1 - \left(1-e_i\right)^\gamma w_{t+1}^{1-\gamma}\right] de \\ &= \left(w_{t+1}^{-\delta} - w_t^{-\delta}\right) - \frac{1}{1+\gamma} w_{t+1}^{1-\gamma} \left(w_{t+1}^{-\delta(1+\gamma)} - w_t^{-\delta(1+\gamma)}\right). \end{aligned}$$

This can again be written in terms of $x_t = w_t^{-\delta}$, which gives

$$R\left(x_t, x_{t+1}\right) = \left(x_{t+1} - x_t\right) - \frac{1+\delta}{2+\delta} x_{t+1}^{-\frac{1}{1+\delta}} \left(x_{t+1}^{\frac{2+\delta}{1+\delta}} - x_t^{\frac{2+\delta}{1+\delta}}\right).$$

Now that we have determined aggregate regret, all that remains is to specify a function $H\left(.\right)$. In this paper we will make use of the so-called logistic distribution (c.f. Brock and Hommes, 1997)

$$n_{t+1} = \frac{1}{1 + \exp\left[-\beta\left(R\left(x_{t+1}, x_t\right) - C\right)\right]}.$$

3.2.3 The full model: theoretical and numerical analysis

The full model is now given by

$$\begin{aligned} x_{t+1} &= G\left(x_t, n_t\right) = \frac{1-(1-n_t)x_t}{\alpha^\delta + n_t} \qquad (7) \\ n_{t+1} &= H\left(x_t, n_t\right) = \frac{1}{1 + \exp\left[-\beta\left(R\left(G\left(x_t, n_t\right), x_t\right) - C\right)\right]}. \end{aligned}$$

The steady state and the local stability properties of the steady state are discussed in the following proposition, which is a corollary to Proposition 1.

Proposition 2 *Consider dynamical system (7). The steady state is given by* $(x^*, n^*) = \left(\frac{1}{1+\alpha^\delta}, \frac{1}{1+\exp(\beta C)}\right)$*. For* $\alpha > 1$*, this steady state is always locally stable. For* $\alpha < 1$*, the steady state is locally stable (unstable) for* βC *smaller (larger) than* $(\beta C)^* = \ln\left(\frac{1+\alpha^\delta}{1-\alpha^\delta}\right)$.

The Appendix contains simulation results for this model. The left hand column gives graphs for the model with naive and rational agents, as discussed in this section. The first four rows (Figures 1, 3, 5 and 7) show time series on x, the fraction of rational agents, wages for high-skilled labor and enrollments, respectively. All four variables undergo perpetual endogenous fluctuations. Although the model is deterministic, no exogenous shocks are needed to generate persistent ups and downs. This is due to the nonlinear nature of the model. At the steady state labor supply and demand intersect such that an 'unstable cobweb' arises. However, if regret becomes 'too large' agents choose a rational predictor which dampens the cycle. The time series on the fraction of rational forecasters shows sharp peaks when the stabilizing force comes into play.

The attractors (rows five to seven, Figures 9, 11 and 13) illustrate the long run behavior of the model. For that purpose we iterated the system 1000 times and dropped the first 100 values. For a stable steady state the figures would show a single point, the steady state value. The unstable case is characterized by enrollments and wages moving over some nontrivial *attractor*. The shape of the attractor depends upon whether we show enrollments over current wages, lagged wages, or wages one period ahead. This indicates that one would find different relationships between enrollments and relative wages when confronted with such a data set, depending on how agents' expectations are modelled empirically.

The bifurcation diagrams capture the dependence of the long-run states of the model on the parameters. In all figures we varied the parameters that are key for the fraction of rational agents (β and C). As β increases the number of rational forecasters in the steady state decreases. From the analysis above we know that, for a sufficiently large value of β an eigenvalue crosses the unit circle. That point corresponds to the value of β where the system undergoes a flip bifurcation. A period two cycle emerges from the stable steady state. As β increases even more, a cascade of period-doubling bifurcations occurs on the route to chaos. Figures 15 and 17 show the bifurcation diagrams for x and n when the information access parameter C is varied. As C increases, it becomes more costly for agents to collect information on the returns to education and consequently the fraction of rational agents in the steady state decreases, which will eventually destabilize the steady state equilibrium. In that case there are too many agents who use past returns on education to predict future returns on which they base their schooling decision and a period doubling bifurcation takes place. As C increases further, more period doubling bifurcations occur again leading to complicated endogenous fluctuations around the steady state. For relatively high values of C the steady state seems to be stable again. A possible explanation for this might be that, for large C, the systems gets very close to the steady state and due to a lack of accuracy in the computer program the system gets stuck in this steady state. Note, that the system is unstable for those parameters. Therefore, a small deviation from the steady state would lead to diverging trajectories (contrary to the stable case where the system would always move back to the steady state after a small deviation).

The last two bifurcation diagrams (figures 23 and 25) show the long-run behavior of the variables x and n as a function of β and C, respectively, in a single figure. The multi-dimensional bifurcation diagrams merge the information of the figures 19 and 21 as well as 15 and 17 in the left hand column, respectively.

4 A variation on the example: naive versus steady state forecasters

It might be argued that rational expectations is a very demanding assumption. Therefore we study other types of predictors. In this section we will briefly look at an example where the rational agents are substituted by agents who compute and predict the steady state w^* as the next wage. This assumption is less strict on what agents know about the future state of the labor market. In fact, it only requires that agents find out about the steady state wage differentials between high-skill and low-skill jobs. In the following subsections we will discuss the ramifications for the general setup and for the numerical example.

4.1 General setup

Let us denote by n_t the fraction of steady state forecasters and by $1-n_t$ the fraction of naive agents. Furthermore, we will assume that steady state forecasters have to pay information costs C in order to find out the steady state, or the long run wage differential between high-skilled and low-skilled work.

With steady state agents instead of rational agents market equilibrium (1) becomes

$$l^d(w_{t+1}) = (1-n_t)F(e(w_t)) + n_t F(e(w^*)), \tag{8}$$

which again implicitly defines w_{t+1} as a function of w_t and n_t, say $w_{t+1} = G_s(w_t, n_t)$.

The fraction of steady state agents depends upon their forecasting performance relative to the naive agents. As long as the economy is in its steady state, the former will always have made the correct schooling decision. However, contrary to rational agents, they may regret their human capital investment or their decision not to invest if the labor market undergoes fluctuations. Then, it may happen, quite similar to the naive forecasters, that the actual wage was above their predicted wage, so that some agents would rather have invested into schooling. Or, if the actual wage falls short of the steady state wage, some agents regret investing into schooling ex post. Therefore, aggregate regret for steady state agents from period t is given by

$$R(w^*, w_{t+1}) = \int_{e(w_{t+1})}^{e(w^*)} R(e, w_{t+1})\, dF(e).$$

Moreover, for naive expectations we have

$$R(w_t, w_{t+1}) = \int_{e(w_{t+1})}^{e(w_t)} R(e, w_{t+1})\, dF(e).$$

As in the previous example we will assume that the fraction of steady state agents will be driven by the difference between aggregate regret of steady state agents and naive agents. This difference is

$$\begin{aligned} R^d(w^*, w_t, w_{t+1}) &= R(w_t, w_{t+1}) - R(w^*, w_{t+1}) \\ &= \int_{e(w^*)}^{e(w_t)} R(e, w_{t+1})\, dF(e). \end{aligned}$$

The updating of fractions follows

$$n_{t+1} = H\left(\beta\left(R^d(w_t, w^*) - C\right)\right),$$

and the full model now becomes

$$\begin{aligned} w_{t+1} &= G_s(w_t, n_t) \\ n_{t+1} &= H\left(\beta\left(R^d(w_t, w^*) - C\right)\right). \end{aligned} \tag{9}$$

The next proposition gives the steady state and its local stability properties.

Proposition 3 *The steady state of dynamical system (9) is*

$$(w^*, n^*) = (w^*, H(-\beta C)).$$

This steady state is locally stable if $\left|l_w^d\right| \geq l_w^s$. *If* $\left|l_w^d\right| < l_w^s$, *there exists a critical value* $(\beta C)^*$ *of* βC *such that for* $\beta C < (\beta C)^*$ *the steady state is locally stable and for* $\beta C > (\beta C)^*$ *the steady state is unstable. Furthermore,* $(\beta C)^*$ *is implicitly given by*

$$H\left(-(\beta C)^*\right) = \frac{l_w^s - \left|l_w^d\right|}{l_w^s}.$$

Proof. The steady state follows directly from (9) and the observations that, at a steady state, $w_t = w^*$ and $R^d(w^*, w^*) = 0$. To find the Jacobian matrix, let us totally differentiate the market equilibrium condition (8) to obtain

$$\left.\frac{\partial G_s}{\partial w_t}\right|_{(w^*,n^*)} = (1 - n^*)\frac{l_w^s}{l_w^d} \text{ and } \left.\frac{\partial G_s}{\partial n_t}\right|_{(w^*,n^*)} = 0.$$

Furthermore, we have

$$\left.\frac{\partial n_{t+1}}{\partial n_t}\right|_{(w^*,n^*)} = \beta H'(-\beta C)\left.\frac{\partial G}{\partial n_t}\right|_{(w^*,n^*)} = 0.$$

The eigenvalues of the Jacobian matrix, evaluated at the steady state, are therefore

$$\lambda_1 = (1 - n^*)\frac{l_w^s}{l_w^d} \text{ and } \lambda_2 = 0.$$

Thus, the steady state is locally stable whenever $(1 - n^*)\, l_w^s < \left|l_w^d\right|$. This is the case when $\left|l_w^d\right| \geq l_w^s$. However, if $\left|l_w^d\right| < l_w^s$, then for all

$n^* < \frac{l_w^s - \left|l_w^d\right|}{l_w^s}$, the first eigenvalue is smaller than -1, and the steady state is unstable. ■

Notice that there is a subtle difference between the bifurcation value for βC in the above proposition and the bifurcation value for the case of rational versus naive agents as given in Proposition 1.

4.2 Numerical example

As before we consider a Cobb-Douglas utility function and a uniform distribution of effort costs. The market equilibrium condition (8) becomes

$$\left(\frac{\alpha}{w_{t+1}}\right)^{\frac{1}{1-\mu}} = (1-n_t)\left(1-w_t^{-\delta}\right) + n_t\left(1-(w^*)^{-\delta}\right).$$

Notice that we can explicitly determine w_{t+1} as a function of w_t and n_t for all admissable values of δ and μ. However, in order to be able to compare this case with the case of rational versus naive agents we will maintain the assumption $\delta(1-\mu) = 1$. We then define $x_t = w_t^{-\delta}$ again and find[4]

$$x_{t+1} = \frac{1}{\alpha^\delta}(1-n_t)(1-x_t) + \frac{1}{1+\alpha^\delta}n_t. \tag{10}$$

For the updating of the fraction of steady state agents we compute the difference in aggregate regret

$$\begin{aligned} R^d(w_t, w_{t+1}) &= \int_{e(w^*)}^{e(w_t)} R(e, w_{t+1})\, dF(e) \\ &= \int_{1-(w^*)^{-\delta}}^{1-(w_t)^{-\delta}} \left[1-(1-e_i)^\gamma w_{t+1}^{1-\gamma}\right] de \\ &= \left((w^*)^{-\delta} - w_t^{-\delta}\right) - \frac{1}{1+\gamma} w_{t+1}^{1-\gamma}\left((w^*)^{-\delta(1+\gamma)} - w_t^{-\delta(1+\gamma)}\right). \end{aligned}$$

[4] Just like in the previous section, we should not allow the wage in sector H to fall below 1. By a similar reasoning as before, the actual development of the variable x_{t+1} can be shown to be represented by

$$x_{t+1} = \min\{\frac{1}{\alpha^\delta}(1-n_t)(1-x_t) + \frac{1}{1+\alpha^\delta}n_t, 1\}.$$

Rewritten in terms of x_t this gives

$$R^d(x_t, x_{t+1}) = (x^* - x_t) - \frac{1+\delta}{2+\delta} x_{t+1}^{-\frac{1}{1+\delta}} \left((x^*)^{\frac{2+\delta}{1+\delta}} - x_t^{\frac{2+\delta}{1+\delta}} \right).$$

The full model becomes

$$\begin{aligned} x_{t+1} &= G_s(x_t, n_t) = \frac{1}{\alpha^\delta}(1-n_t)(1-x_t) + \frac{1}{1+\alpha^\delta} n_t \qquad (11) \\ n_{t+1} &= H(x_t, n_t) = \frac{1}{1+\exp\left[-\beta\left(R^d(x_t, G_s(x_t, n_t)) - C\right)\right]}. \end{aligned}$$

The following result is a straightforward application of Proposition 3.

Proposition 4 *Consider dynamical system (11). The steady state is given by* $(x^*, n^*) = \left(\frac{1}{1+\alpha^\delta}, \frac{1}{1+\exp(\beta C)}\right)$. *For* $\alpha > 1$, *this steady state is always locally stable. For* $\alpha < 1$, *the steady state is locally stable (unstable) for* βC *smaller (larger) than* $(\beta C)^* = \ln\left(\frac{\alpha^\delta}{1-\alpha^\delta}\right)$.

The right hand column in the Appendix shows simulations of the model with steady state forecasters. All parameters are equal to the simulations for the model with rational agents to assure comparability. A brief look already reveals that the qualitative dynamic properties do not change. The time series on wages and enrollments still undergo perpetual endogenous fluctuations. For most of the time agents use the naive prediction rule. Those periods of naive forecasts are interrupted by sharp peaks where the fraction of steady state forecasters increases.

The attractors are stretched compared to the the attractors of the numerical example with rational agents. But their qualitative appearance does not change with steady state forecasters instead of rational agents.

The bifurcation diagrams repeat the analytical results derived from the eigenvalues of the linearized model. There, we already demonstrated that the eigenvalue crosses the unit circle for lower values β and C, respectively. This can be seen in the bifurcation diagrams, where for lower values of β and C the first flip bifurcations occurs.

5 Concluding remarks

Our aim was to explain variations in enrollments to higher education endogenously. For that purpose we developed a human capital model with overlapping generations. In this model agents have the choice to invest into schooling in the first period of their life to earn a higher wage in the second period of their life, or not to put effort into schooling and work in the low-skill sector in both periods. Contrary to other human capital models our agents are heterogenous in their expectations on the returns to education. As access to information is costly, they use current returns on education as a predictor for future returns, unless experience of the previous generation indicates that using a more sophisticated prediction rule is advantageous. Hence, intergenerational spill-overs lead the (naive) predictor choice. The interplay of destabilizing backward looking expectations and a stabilizing more sophisticated predictor may generate endogenous fluctuations in the demand for education. No exogenous shock are needed to arrive at continuing changes in enrollments. This holds true, even under standard assumptions on labor demand, which is downward sloping in our examples, and agents' preferences, that are Cobb-Douglas. We illustrated our point for two different sophisticated prediction rules: rational expectations, and agents who know the steady state wage differential between high-skill and low-skill jobs. No matter, which sophisticated prediction rule we choose, the qualitative results stay.

The distribution of agents that react to regret, a function that captures the previous generations' extent of dissatisfaction about schooling decisions, is key for the dynamics of our model. This suggests, that policies tearing down obstacles for collecting information on returns to education may stabilize flows to higher education. The reason is, that students will less likely make use of cheap and possibly destabilizing backward looking prediction rules. If it is more easy to find and process information on future labor market states, more students will be guided by the 'true' wage differentials in their schooling choice.

The model that we developed is parsimonious, mainly for reasons of tractability. Extensions of the model may include drop-outs from school, and possibly a more sophisticated life-time structure. Empirically, it would be interesting to have time series evidence on agents' expectation formation, to see whether it resembles the variation over time generated by our model.

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