

Exzerpt from: “Basic Math for Social Scientists. Concepts”

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1 Exponents

1. $a = a^1$

2. $a^m \times a^n = a^{m+n}$

3. $(a^m)^n = a^{m \times n}$

4. $(a \times b)^n = a^n \times b^n$

5. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

6. $a^{-n} = \frac{1}{a^n}$

7. $\frac{a^m}{a^n} = a^{m-n} = \frac{1}{a^{n-m}}$

8. $a^{\frac{1}{2}} = \sqrt{a}$

9. $a^{\frac{1}{n}} = \sqrt[n]{a}$

10. $a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = (a^m)^{\frac{1}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

11. $a^0 = 1$

12. $0^0 = \{\}$

2 Logarithms

1. $\log_a a^x = x$

2. $a^{\log_a x} = x$

3. $\log(a \times b) = \log a + \log b$

4. $\log\left(\frac{a}{b}\right) = \log a - \log b$

5. $\log(a^n) = n \log a$

3 Factors

1. $n! = n \times (n-1) \times (n-2) \times \dots \times 1$

2. $0! \stackrel{\text{def}}{=} 1$

4 Intervalls

1. $[a, b] = \{x | a \leq x \leq b\}$
2. $(a, b) = \{x | a < x < b\}$
3. $(a, b] = \{x | a > x \leq b\}$

5 Permutations and Combinations

1. If sets A_1, A_2, \dots, A_m have respectively, n_1, n_2, \dots, n_m elements, then there are $n_1 \times n_2 \times \dots \times n_m$ ways of selecting one element from each set.
2. The number of permutations (order does matter) that can be formed by selecting r objects from a set of n objects is

$$P(n, r) = \frac{n!}{(n-r)!}$$

3. The total number of different permutations of n objects of which r_1 are the same, r_2 others are the same, \dots , and r_j others are the same is

$$\frac{n!}{r_1! \times r_2! \times \dots \times r_j!}$$

4. The number of combinations (order does not matter) of r objects taken from a set of n objects is

$$\binom{n}{r} = \frac{n!}{r! \times (n-r)!}$$

6 Limits

Suppose that $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ both exist. Then,

1. $\lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$
2. $\lim_{x \rightarrow a} [f(x) \times c] = c \times \lim_{x \rightarrow a} f(x)$
3. $\lim_{x \rightarrow a} [f(x) \times g(x)] = \lim_{x \rightarrow a} f(x) \times \lim_{x \rightarrow a} g(x)$
4. $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$
5. $\lim_{x \rightarrow a} f(x)^n = \left[\lim_{x \rightarrow a} f(x) \right]^n$

7 Differential Calculus

The derivative is defined as the slope of a function $f(x)$ at point x . That is,

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{(f(x) + \Delta x) - f(x)}{\Delta x} = \frac{dy}{dx}$$

7.1 Differentiation Rules

1. $\frac{d}{dx}a = 0$
2. $\frac{d}{dx}a + bx = b$
3. $\frac{d}{dx}g(x) + h(x) = g'(x) + h'(x)$
4. $\frac{d}{dx}x^n = nx^{n-1}$
5. $\frac{d}{dx}\sin x = \cos x$
6. $\frac{d}{dx}\cos x = -\sin x$
7. $\frac{d}{dx}\log x = \frac{1}{x} = x^{-1}$
8. $\frac{d}{dx}e^x = e^x$
9. $\frac{d}{dx}g(x)h(x) = g'(x)h(x) + g(x)h'(x)$
10. $\frac{d}{dx}\frac{g(x)}{h(x)} = \frac{g'(x)h(x) - g(x)h'(x)}{h(x)^2}$
11. $\frac{d}{dx}g(h(x)) = g'(h(x))h'(x)$

7.2 Partial Derivatives

For functions with more than one variable usually the notation $\frac{\partial}{\partial u}$ where u denotes the variable we differentiate.

7.3 Extremas

1. A local Minimum of $\frac{d}{dx}$ is a point where $f'(x) = 0$ and $f''(x) > 0$
2. A local Maximum of $\frac{d}{dx}$ is a point where $f'(x) = 0$ and $f''(x) < 0$